Economics Tripos Part 1 Paper 3
Quantitative Methods in Economics
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Supervision 2 (mathematics): constrained optimisation

Readings: Chiang and Wainwright ch. 11, 12

Questions

1. Consider an objective function $Z = -100 + 80X - 0.1X^2 + 100Y - 0.2Y^2$.

a) Solve for the values $(X*, Y*)$ which maximise Z.

b) Now suppose there is a constraint, $X + Y = 325$. Maximise Z subject to this constraint and find the corresponding values of $(X*, Y*)$.

c) Does the constraint bind?

d) Assume the constraint equation changes to $X + Y = 400$. Find the new solution. Does the constraint bind?

2. A consumer has utility function $U(x_1, x_2) = x_1x_2 + 2x_1 + 5x_2$. She has a quantity $a > 0$ of good 1 and none of good 2. The price of good i $(i = 1, 2)$ is given by p_i.

a) Write down the budget constraint of the consumer and derive the first order conditions of the optimisation problem using Lagrange.

b) Compute the optimal consumption quantities (x_1*, x_2*).

c) How much of good 1 does the consumer sell in the market? Check that the value of what he sells is equal to the value of what he buys.

d) Setting $p_1 = 1$, compute the effect on x_1*, x_2* and $U(x_1*, x_2*)$ of a marginal increase in a.

3. A firm produces goods x and y with (joint) cost function $c(x, y) = 6x^2 + 10y^2 - xy + 30$ and production quota $x + y = 34$.

a) Which combination of goods should the firm produce to minimise costs?

b) Estimate the effect on costs if the production quota is reduced by one unit.

4. A firm has total profit function $\pi(x, y) = 80x - 2x^2 - xy - 3y^2 + 100y$ and its maximum output capacity is $x + y = 12$.

a) What output mix should it produce?

b) Estimate the effect on profit if output capacity expands by one unit.

c) Does the constraint bind?

5. A rancher faces the profit function $\pi(x, y) = 100x - 3x^2 - 2xy - 2y^2 + 140y$ where x =sides of beef and y =hides. There are two sides of beef for every hide.

a) How would you express the constraint in terms of x and y?

b) At what level of output will the rancher maximise profits subject to the constraint?

6. A monopolistic firm sells two products, x and y, with the following demand functions:

$x = 72 - 0.5P_x$ $\qquad\qquad$ $y = 120 - P_y$

The combined cost function is $c(x, y) = x^2 + xy + y^2 + 35$ and maximum joint production is 40 i.e. $x + y = 40$. Find the profit maximising level of:

a) Output

b) Price

c) Profit

d) Does the constraint bind? What would be the profit maximising level of output, price and profit if there was no output constraint?

7. Tripos 2005 A1

8. Tripos 2006 A3

9. Tripos 2007 B2

10. Tripos 2008 4A

11. Tripos 2009 B2 (except for last bit about Cramer's rule)