

Economics Tripos Part 1 Paper 3
Quantitative Methods in Economics
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Supervision 4 (statistics): test procedures; confidence intervals; statistical inference

Questions

1. Use the standard normal distribution to find the following probabilities:

- a) $P(Z > 1.96)$
- b) $P(Z < 0.1)$
- c) $P(-2.9 < Z < 1.9)$
- d) $P(Z < -0.1)$
- e) $P(0.1 < Z < 0.96)$

2. Suppose you repeatedly select random samples from a population and calculate their means. The Central Limit Theorem states that the sampling distribution of the mean of a variable x :

- is normally distributed
- has mean equal to the population mean
- has variance equal to population variance divided by sample size n

This implies that $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$.

The conditions for the CLT to hold are that:

- samples are sufficiently large and of equal size
- sampling is carried out with replacement

State whether you think the CLT applies in the following situations and explain your reasoning.

- a) The wing span of 15 swans observed on the Cam river.
- b) The colour of a ball drawn from an urn of 100 mixed black and white balls.
- c) The height of pine trees chosen as Christmas trees in the UK in 2005.

3. What do you understand by the following terms?

- a) The null hypothesis.
- b) Type I and Type II errors.
- c) The significance level of a test.
- d) The power of a test.

4. In a population of baby girls the weight is normally distributed with $\mu = 3480g$ and $\sigma = 1400g$. Given a random sample of size $n = 50$, what is the probability that the weight is:

- a) greater than $4000g$
- b) less than $3500g$
- c) between $3000g$ and $4000g$

5. Waiting time in the Trinity Street post office is on average 20 minutes with a variance of 25 minutes. A research study interviews 50 customers. What is the probability that a customer's waiting time is:

- a) less than 18 minutes
- b) more than 22 minutes
- c) What is the upper bound for waiting time if the probability is 95%?
- d) What is the minimum time a customer would have to wait with probability 90%?

6. In the Czech Republic, 9% of apples grown have a darker colour than the rest. Randomly draw a sample of 300 apples. If f is the proportion of darker apples, find:

- a) $\Pr(0.08 < f < 0.09)$
- b) $\Pr(f < 0.07)$.

7. We are interested in the average income of a non-EU worker in Cambridge. From the population of all non-EU workers in Cambridge, we draw a random

sample of 100 workers. In this sample the mean income is £500 and the standard error is 100.

a) Determine the confidence interval of our sample estimation with error 95%. Comment on the width of the confidence interval and the absolute error we would expect.

b) How big should the sample be if we want an error of around 10?

c) Construct the confidence interval for the case $n = 20$.

8. You are asked to perform a hypothesis test about a population mean μ from a population with known variance σ^2 . Which of the following assumptions are needed to use the mean of the data, \bar{x} , to perform the test? Justify your answer.

I. The data represent a random sample from the population.

II. The population is normally distributed.

III. The sample size is large.

9. What is the difference between a one-sided and two-sided hypothesis test?

10. In which situations is a Student's t distribution used instead of the normal distribution?

11. Suppose X is normally distributed with mean μ and variance σ^2 . We collect data on a sample of size $n = 50$ and find that $s^2 = 144$. Determine a 90% confidence interval for σ^2 .

12. The sample mean is $\bar{x} = 900$ with sample standard deviation $s^2 = 100$ and $n = 20$.

a) Test $H_0 : \mu = 1000$ against the alternative $H_1 : \mu \neq 1000$ with $\alpha = 0.05$.

b) Change the alternative to $H_1 : \mu = 880$ and perform the test again.

c) 6 more observations are added to the sample but the sample variance does not change. Do the results of your tests change? Explain.

13. The quantity of sugar contained in a certain product is analysed. A sample of 6 packs gives the following results:

16.6, 17.1, 17.4, 16.8, 16.4, 17.1

a) Test $H_0 : \sigma^2 = 1$ at significance level $\alpha = 0.02$.

b) More observations are added to the sample but the sample variance does not change. Does the result of your test change? Explain.

c) What happens if we change the null hypothesis to $H_0 : \sigma^2 = 0$?

14. Tripos 2005 C2

15. Tripos 2006 C4

16. Tripos 2006 C6

17. Tripos 2007 C5

18. Tripos 2008 C1

19. Tripos 2008 C6