Economics Tripos Part 1 Paper 3
Quantitative Methods in Economics
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Supervision 5 (mathematics): Integration, matrix algebra, determinants, inverses of matrices, Cramer's rule, definiteness of matrices, concavity properties of functions

Readings: Chiang and Wainwright ch. 4-5, 14. A more advanced reference for matrices is Simon and Blume.

Questions

1. Find the integrals of the following functions:

(a) $\int \left(x^3 + x + 1\right) dx$

(b) $\int (2e^x + \frac{14x}{7x^2+5})dx$

(c) $\int 2x(x^2 + 1)dx$

(d) $\int 2^{3x} dx$

(e) $\int 12x^2(x^3 + 2)dx$

(f) $\int_0^3 4e^{2x} dx$

(g) $\int_0^2 \frac{3x^2}{(x^3+1)^2} dx$

2. Consider the following matrices.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 9 & 1 \\ 9 & 8 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 6 \\ 9 & 1 \\ 0 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & -3 & -1 \\ 9 & 2 & 8 \end{bmatrix}$$

(a) What are the dimensions of each matrix?

(b) Compute the following operations. If they cannot be performed, explain why not.

i. $2A$ ii. $B+D$ iii. D' (or D^T; both mean the transpose) iv. $B+D'$ v. $\det C$ vi. $D*B$ vii. $B*D$ viii. $C*D$ ix. $D*C$ x. $A*B$ xi. $B*A$ xii. C^{-1} xiii. $\det A$ xiv. $\det B$

3. Calculate the determinants of the following matrices. Which ones are singular?

(a) $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 0 & 9 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

4. Put the following systems of equations into matrix form and find the solutions using Cramer's rule.

(a)
$$5x + 10y = 200$$
$$x - y = 50$$

(b)
$$x_1 + 3x_2 = 0$$
$$2x_1 + 4x_2 = 6$$

(c)
$$4a + 26 - 3c + 12 = 0$$
$$5a - 5b - 10 = 0$$
$$6c + 5b = -2c$$

(d)
$$3q + 10p - 2y = 100$$
$$21 + 10y - 3p = 50$$
$$y + p + q = 0$$

5. Tripos 2006 A1

6. Tripos 2007 A1

7. The simple IS-LM model of the macroeconomy consists of two equations

$$Y = cY - \beta r + G, \qquad 0 < c < 1, \beta > 0$$
$$M = \alpha Y - \lambda r, \qquad \alpha > 0, \lambda > 0$$

(a) Write down this model as a matrix equation.

(b) Use Cramer's rule to solve for r as a function of the exogenous variables G and M.

8. Classify the following matrices as positive definite, positive semi-definite, indefinite, negative semi-definite or negative definite:

(a) $\begin{bmatrix} 5 & 2 \\ 4 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ 4 & -3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 10 \\ 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 2 \\ -2 & 0 \end{bmatrix}$

9. Find the stationary points of these functions and classify them by checking the definiteness of the Hessian matrix.

(a) $f(x, y) = x^3 + xy^2 - 12x - y^2$

(b) $f(x, y) = 3x^2 - 6xy + y^2 + y^4$

10. (a) Define concavity of a function $f(x, y)$ in terms of the Hessian matrix.

(b) Define convexity of a function $f(x, y)$ in terms of the Hessian matrix.

(c) Classify the following functions as convex or concave:

i. $f(x, y) = 3x^2 + y^2$

ii. $f(x, y) = -x^2 - 4y^2$

iii. $f(x, y) = 2 + 5x - 3y^2$

iv. $f(x, y) = 2x^3 + 7y^2$

11. Tripos 2009 A1