Economics Tripos Part 1 Paper 3
Quantitative Methods in Economics
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Supervision 7 (mathematics): Further work with matrices, differential and difference equations, Taylor series

Readings: Simon and Blume, Alpha C. Chiang

Questions

1. Tripos 2009 B1

2. Tripos 2008 A3

3. Assume that the utility function $U(x_1, x_2)$ is a concave function of (x_1, x_2). Define R, the set of commodity bundles which are at least as good as the bundle $(\bar{x}_1, \bar{x}_2)$, as $R = \{(y_1, y_2) \mid U(y_1, y_2) \geqslant U(\bar{x}_1, \bar{x}_2)\}$.

Show that R is a convex set. (Hint: R is the set of all commodity bundles which lie on or above the indifference curve passing through $(\bar{x}_1, \bar{x}_2)$).

4. Use the Taylor series expansion formula to find an estimate for the function $f(x) = e^x$, for any value belonging to the interval $[0, 1]$. Choose $x_0 = 0$ and ensure that your computation is correct to within 0.001.

5. Compute the fourth-order Taylor approximation of $f(x) = \frac{1}{1-x}$ around $x = 0$.

6. For each of the following difference equations,

i. obtain the general solution,

ii. obtain the particular solution,

iii. and solve for the steady state, if it exists, and indicate whether or not y_t converges to the steady state.

(a) $y_{t+1} = 2y_t - 10$

1

(b) $y_{t+1} = y_t$

(c) $y_t = 0.5y_{t-1} + 1$

7. For a given y_0, solve $y_{t+1} = \alpha^t y_t + b$.

8. Solve the following linear, first-order differential equations, making sure the initial conditions are satisfied.

(a) $\dot{y} - y = 0$, $\qquad y(0) = 1$

(b) $\dot{y} + 3y = 12$, $\qquad y(0) = 10$

(c) $2\dot{y} + \frac{1}{2}y = 12$, $\qquad y(0) = 10$

(d) $\dot{y} = 5$, $\qquad y(0) = 1$

(e) $\dot{y} - 6y = 0$, $\qquad y(0) = 3$.

9. Consider the non-linear difference equation:

$$x_{t+1} = f(x_t) = x_t^2.$$

(a) Find the unique strictly positive stationary point of $\bar{x}$ of this equation.

(b) Transform this equation into an approximately linear equation by taking a first-order Taylor approximation of $f(x)$ around $x = \bar{x}$.

(c) Analyse the stability of the approximated difference equation.

10. Expressions for demand and supply in the market for coffee are given by:

$$D_t = a_1 - a_2 P_t$$
$$S_t = b_1 + b_2 P_t^e$$

P_t^e denotes past expections of P_t (i.e. expectations made at $t-1$ about what the price will be at t). The current price P_t always adjusts to clear the market, hence $S_t = D_t$.

(a) Calculate the price of coffee in long-run equilibrium (assuming existence of a long-run equilibrium). Comment on the stability of this equilibrium.

(b) Suppose that coffee bean growers expect next period's price to be equal to the current period's price, i.e. $P_t^e = P_{t-1}$. Find an expression for the price of coffee at time t, given that price at time 0 is P_0.

(c) How does the price change over time? Are the dynamics dependent on the values of the parameters? If so, give details of the different cases.

(d) How accurate is the expectation $P_t^e = P_{t-1}$ in the short run? And in the long run? How might a "rational" coffee bean grower form his expectations to avoid making errors?

11. Tripos 2006 B2

12. Tripos 2004 A5

13. Tripos 2009 A3

14. Tripos 2009 A4